

Society, I should be glad if you could find space in NATURE for an account of them.

The dust consisted essentially of ferruginous sand, chalk, and silicates of alumina, alkalis, lime and magnesia, mixed with a certain quantity of organic matter and with an appreciable proportion of lead.

The last-named substance is probably due to the sample having been collected from a leaded roof. It may either have been scraped off during the taking of the sample, or, possibly, cut from the leads by the impact of sand particles driven against the roof by a high wind. Traces of tin and arsenic were also present in the sample; these were probably contained as impurities in the lead.

The detailed results of the analysis are as follows:—

(Substance dried at 100° C. before analysis.)

	Per cent.
Loss on heating to redness	11.28
Lead, calculated as oxide	3.31
Arsenic	0.01
Tin	Traces

After deducting the lead, tin and arsenic as being probably adventitious, the remainder of the sample is made up of the following constituents:—

	Per cent.
Silica	45.94
Alumina	18.35
Iron oxide	6.57
Lime	8.64
Magnesia	1.86
Alkalis { Sodium oxide	1.16
{ Potassium oxide	2.30
Carbonic acid	6.10
Water and organic matter	9.08
	100.00

The organic matter contained 2.19 per cent. of carbon and 0.16 per cent. of nitrogen, the two representing, probably, between 3 and 4 per cent. of organic constituents.

After being heated to redness, 33.30 per cent. of the sample was found to be soluble in hydrochloric acid, the dissolved portion including practically the whole of the lead, with the traces of tin and arsenic. Again deducting those elements, the dissolved constituents were as follows:—

	Per cent.
Silica	0.64
Alumina	11.20
Iron oxide	5.43
Lime	8.19
Magnesia	1.13
Alkalis	1.46
Carbonic acid	3.48
	31.53

Thus about one-third of the sample is dissolved by hydrochloric acid, including the greater part of the alumina, iron, lime and magnesia, but only a small fraction of the silica.

Dilute acetic acid readily dissolved out the greater part of the lime, with liberation of carbonic acid gas. Water alone dissolved practically nothing from the sample except minute traces of lime. These results show that most of the lime is present in the sample in the form of chalk.

One or two particles of metallic lead were detected in the sample, together with others partly oxidised and carbonated.

It has been surmised by Dr. Mill and others that the sand which accompanied the storm of February 22, and was observed to fall in a great number of places in this country as well as on the Continent, was originally derived from the African deserts.

It would be interesting in this connection to compare its characters with that of the dust, also presumably of African origin, which was observed to fall in the neighbourhood of Taormina, by Sir Arthur Rücker, and was made the subject of an interesting communication to NATURE by Prof. Judd about a year ago.

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The Undistorted Cylindrical Wave.

THE receipt of a paper by Prof. H. Lamb, "On Wave Propagation in Two Dimensions" (*Proc. Lond. Math. Soc.*, vol. xxxv. p. 141), stimulates me to publish now a condensation of a portion of a work which will not be further alluded to. I once believed that there could not be an undistorted cylindrical wave from a straight axis as source. But some years ago the late Prof. FitzGerald and I were discussing in what way a plane electromagnetic wave running along the upper side of a plane conducting plate, and coming to a straight edge, managed to turn round to the other side. Taking the wave as a very thin plane slab, one part of the theory is elementary. The slab wave itself goes right on unchanged. Now Prof. FitzGerald speculatively joined it on to the lower side of the plate by means of a semi-cylindrical slab wave. I maintained that this could not possibly work, because the cylindrical wave generated at the edge was a complete one, causing backward waves on both sides of the plate. Moreover, it was not a simple wave, for the disturbance filled the whole cylindrical space, instead of being condensed in a slab. It was in the course of examining this question that I arrived at something else, which I thought was quite a curiosity, namely, the undistorted cylindrical wave.

Maxwell's plane electromagnetic wave consists of perpendicularly crossed straight electric and magnetic forces, in the ratio given by $E = \mu v H$. Thinking of a thin slab only, it travels through the ether perpendicularly to itself at speed v , without any change in transit. I have shown that this may be generalised thus. Put any distribution of electrification in the slab, and arrange the displacement \mathbf{D} in the proper two-dimensional way, as if the medium were non-permittive outside the slab. Then put in \mathbf{H} orthogonally, according to the above mentioned rule, and the result is the generalised plane wave, provided the electrification moves with the wave. Otherwise, it will break up. Another way is to have the electrification upon fixed perfectly conducting cylinders arranged with their axes parallel to the direction of propagation.

Now the first kind of plane wave has no spherical analogue, obviously. But I have shown that the other kinds may be generalised spherically. Put equal amounts of positive and negative electrifications on a spherical surface arranged anyhow. Distribute the displacement in the proper way for a spherical sheet, as if constrained not to leave it. Then put in \mathbf{H} orthogonally as above. The result constitutes an undistorted spherical electromagnetic wave, provided the electrification moves radially with the wave, and attenuates in density as its distance from the centre increases, in the proper way to suit \mathbf{E} and \mathbf{H} . This attenuation does not count as distortion. Similarly, the other sort of generalised plane wave may be imitated spherically by having conical boundaries.

But when we examine the cylinder, there is apparently no possibility of having undistorted waves. For with a simple axial source it is known that if it be impulsive, the result is not a cylindrical impulse, but that the whole space up to the wave front is filled with the disturbance. It is easy to see the reason, for any point within the wave front is receiving at any moment disturbances from two points of the source on the axis, and there is no cancellation. And if the source be on a cylindrical surface itself, producing an inward and an outward wave, the whole space between the two wave fronts is filled with the disturbance.

How, then, is it possible to have an undistorted wave from a straight line source? By not arguing about it, but by showing that it can be done. The reason will then come out by itself. As the solution can be easily tested, it is only necessary to give the results here. Take plane coordinates r and θ . Let the magnetic force be perpendicular to the plane, of intensity H . Let Z be its time-integral, then

$$Z = \frac{\cos \frac{1}{2}\theta}{vr^{\frac{1}{2}}} f(vt - r), \quad H = \frac{\cos \frac{1}{2}\theta}{r^{\frac{1}{2}}} f'(vt - r), \quad (1)$$

expresses the magnetic field, f being an arbitrary function. Now the displacement \mathbf{D} is the curl of \mathbf{Z} . So if E_1 is the radial component of \mathbf{E} , and E_2 the tangential component, in the direction of increasing θ , we have the electric field given by

$$E_1 = \frac{-\mu v \sin \frac{1}{2}\theta}{2r^{\frac{1}{2}}} f, \quad E_2 = \frac{\mu v \cos \frac{1}{2}\theta}{r^{\frac{1}{2}}} f' + \frac{\mu v \cos \frac{1}{2}\theta}{2r^{\frac{1}{2}}} f. \quad (2)$$

The attenuation factor r^{-1} in (1) does not count as distortion.

The wave may go either way, and various cases can be elaborated. If the wave is outward, the axis ($r=0$) is the source. The plane $\theta=0$ is a perfect electric conductor. The electrification is of the same sign on its two sides. Other details may be got from the formulæ.

I give an example to show the not very obvious electrical meaning. Let the infinite plane conductor with the straight edge be one pole of a condenser, and a straight wire placed parallel to the edge, and close to it, be the other pole. Join them by a battery, charging the plate and the wire. Bring the wire right up to the edge, and reduce its magnitude to a mere line. (This is to be done in order to attain the ideal simplicity of the formulæ.) Take away the battery. Then the electric field is given by

$$c\mathbf{v}E_1 = -\frac{\sin \frac{1}{2}\theta}{2r^{\frac{1}{2}}} f_0, \quad c\mathbf{v}E_2 = \frac{\cos \frac{1}{2}\theta}{2r^{\frac{1}{2}}} f_0, \quad (3)$$

where f_0 is a constant and c is the permittivity.

Finally, discharge the condenser by contact between edge and wire. Then the result at time t later is that outside the cylinder of radius $r=vt$ the above field (3) persists, whilst inside the cylinder there is no \mathbf{E} or \mathbf{H} . An electromagnetic wave separates these regions. It started from the axis at the moment of contact, and as it expands swallows up the whole energy of the field, and carries it to infinity. Similarly, as regards the charging of the plate, only the "battery" should, to have the same formulæ, be an impressed force acting at the axis, between the edge and the wire. At time t after contact, the electric field is established fully within the cylinder $r=vt$. On its boundary is the impulsive wave which is laying down the remainder. It also, if the contact be instantaneous, wastes an equal amount of energy at infinity.

Similarly, by varying the impressed voltage anyhow with the time, the emission of an arbitrary wave of \mathbf{H} results. With a real plate and real wire, the main features would no doubt be the same. The use of the line wire introduces infinite voltage.

What somewhat disguises the electromagnetics is the existence of the steady electric force, or parts thereof, along with the electromagnetic \mathbf{E} and \mathbf{H} , particularly when f is arbitrary. There is a similar complication in the spherical wave when the total electrification in any thin shell is not zero. There is then an auxiliary internal or external electric force to make continuity.

We cannot have an undistorted wave from a simple line source. But in the example the apparent line source will be found to be a doublet. For the curl of \mathbf{e} (impressed force) is the source of the wave. It is double, positive on one side, negative on the other.

Solutions of the type

$$\mathbf{H} = \sum \frac{A r^n \cos(n\theta + a)}{(v^2 t^2 - r^2)^{n+\frac{1}{2}}} \quad (4)$$

or the same with r and vt interchanged in the denominator, are not distortionless, save for the solitary term in which $n=-\frac{1}{2}$. The above distortionless cylindrical wave (1) is unique. Prove by the characteristic.

April 29.

OLIVER HEAVISIDE.

Seismometry and Gëite.

UNDER the above heading Prof. J. Milne contributed an interesting article to NATURE of April 9, p. 538, on which I wish to offer some remarks. Prof. Milne seems hardly to realise the significance of the enormous pressures to which the earth's deep-seated material is presumably exposed. One of his objections to the hypothesis of an iron core seems to be that the wave velocities for an infinite isotropic medium of the density and elasticity of iron do not accord with the velocities of earthquake waves. This objection, however, is not conclusive. In an infinite isotropic medium there are two purely elastic wave velocities, v_1 and v_2 , given by the equations

$$v_1 = \sqrt{(m+n)/\rho}, \quad v_2 = \sqrt{n/\rho},$$

where ρ is the density, m and n Thomson and Tait's two elastic constants. On the ordinary theory, n/m may possess any value consistent with Poisson's ratio γ , or $(m-n)/2m$,

lying between 0 and 0.5. Six years ago I showed (*Phil. Mag.*, March, 1897, p. 199) that observed seismic wave velocities can be accounted for by elastic waves without postulating any abnormal value for Young's modulus—the modulus to which Prof. Milne repeatedly refers. For instance, we get values of 12.5 and 2.5 kilometres per second respectively for v_1 and v_2 in a medium of density 5.5 with a Young's modulus of only 10^9 grammes weight per sq. cm., if we suppose $n/m=1/24$, or $\gamma=0.48$ approximately; and the same results follow if we increase density and elastic constants in the same proportion.

In iron, as we know it, γ , of course, is not 0.48, but more nearly 0.25. A material, however, which under low pressures has $\gamma=0.25$, may, after prolonged exposure to enormous pressures, behave as an elastic medium with γ very nearly 0.5. In fact, if the deep-seated material acts as an elastic medium, the only consistent way yet pointed out for its doing so is by its behaving as if γ were very near the limiting value answering to incompressibility. Neither of the elastic wave velocities, it should be noticed, has anything directly to do with Young's modulus, a point which cannot be too clearly emphasised. Another consideration is the possibly appreciable influence of gravity on the wave velocities.

Coming now to the question of the behaviour of magnetographs at times of seismic disturbance, there must undoubtedly be magnetic disturbances occasioned by earthquakes in more than one way. When a violent earthquake occurs where magnetic material abounds, there may be a vast movement of magnetised matter; there may be a great change in the stresses throughout adjacent magnetic material; and there may be a great change of local temperature. Any one of these causes will give rise to a magnetic disturbance which should be practically simultaneous all over the world, and should precede any seismic movement at distant stations. It should also diminish very rapidly as the distance from the earthquake origin increases.

Again, as the seismic waves travel out from their source they must cross volumes of magnetic matter, and the mechanical effect on any such volume must necessarily produce changes in its magnetic field. Owing to the finite velocity of seismic waves, the displacements and stresses simultaneously existent in different parts of any large magnetic volume must be in all kinds of phases, leading to considerable interference between the magnetic disturbances to which the different parts give rise at any considerable distance. Thus the most plausible explanation of why a magnetic disturbance of some prominence—if real—should appear at one observatory, but not at another only 100 miles off, is certainly the existence of magnetic material close to the former. Supposing that such local material exists, the magnetic phenomena may be expected to vary according to the direction in which the earthquake wave is travelling.

One of the chief difficulties in reaching definite conclusions is the contracted time scale usual in magnetograms. If the true seismic and the apparent magnetic disturbances occur within a few seconds of one another, it is usually practically impossible to say which is the earlier. To see the full force of this, one must remember that a by no means improbable explanation of why apparent magnetic disturbances accompany earthquakes at one station, but not at another, is that the magnets at the former, owing to pattern or site, may be much more sensitive *seismographs* than those at the latter.

Again, it must be remembered that whilst the so-called "large waves"—rather an unfortunate term—produce in general a much greater effect on a horizontal pendulum than do the "preliminary tremors," it by no means follows that the same will be true of either the true magnetic or the purely mechanical effects on a magnet. Much may depend on the method of support and the time of swing.

The passage of the "preliminary tremors" and "large waves" due to an earthquake often occupies several hours, and during this interval several true independent magnetic movements are not at all unlikely to present themselves, even at times of general magnetic calm.

For all these reasons a careful intercomparison is wanted of magnetic and seismic records from a variety of stations. Something might be done by running magnetographs for some time in a district where a local magnetic disturbance